Homework Assignment   Solutions

1. **[Unate Covering] (20 Points)**

   Problem 15 (Chap. 4), p. 172 in Hachtel&Somenzi (H&S).

   Solution: see Figure 1.

2. **[Cofactoring a Cover] (15 Points)**

   Problem 5 (Chap. 5), p. 208 in H&S.

   Solution:

$$\left[\begin{array}{ccccc|ccc} v & w & x & y & z & f & g & h \\ 1 & - & 0 & - & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & - & - & 0 & 0 & 1 \\ - & 1 & 1 & 1 & - & 1 & 1 & 0 \\ 0 & 1 & - & - & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & - & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & - & 1 & 0 & 1 \end{array}\right]_{--110|110} \Rightarrow \left[\begin{array}{ccccc|cc} v & w & x & y & z & f_c & g_c \\ - & 1 & 1 & 1 & - & 1 & 1 & 0 \\ 0 & 1 & - & - & 0 & 1 & 1 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cc|cc} v & w & f_c & g_c \\ - & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{array}\right] = \left[\begin{array}{cc|cc} v & w & f_c & g_c \\ - & 1 & 1 & 1 \end{array}\right]$$

3. **[Tautology Checking] (15 Points)**

   Problem 12 (Chap. 5), p. 210 in H&S.

   Solution: The initial cover can be reduced by eliminating the first cube since it is totally covered by the second cube:

$$\left[\begin{array}{cccc|c} w & x & y & z & f \\ 1 & 1 & 0 & 0 & 1 \\ 1 & - & - & - & 1 \\ - & 0 & - & 1 & 1 \\ 0 & 1 & - & - & 1 \\ 0 & - & 1 & - & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} w & x & y & z & f \\ 1 & - & - & - & 1 \\ - & 0 & - & 1 & 1 \\ 0 & 1 & - & - & 1 \\ 0 & - & 1 & - & 1 \end{array}\right]$$

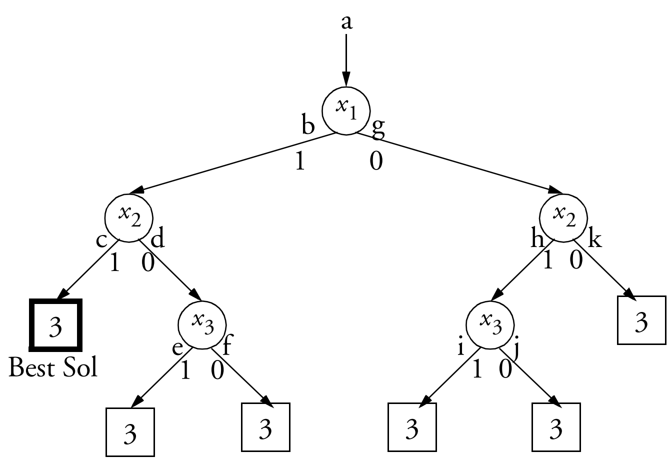
   Since this reduced cover is positive unate in y and z, we can determine if it is a tautology by checking if its cofactor with respect to $y'z'$ is a tautology:

$$\left[\begin{array}{cccc|c} w & x & y & z & f \\ 1 & - & - & - & 1 \\ - & 0 & - & 1 & 1 \\ 0 & 1 & - & - & 1 \\ 0 & - & 1 & - & 1 \end{array}\right]_{--00|1} \Rightarrow \left[\begin{array}{cccc|c} w & x & y & z & f_{y'z'} \\ 1 & - & - & - & 1 \\ 0 & 1 & - & - & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} w & x & f_{y'z'} \\ 1 & - & 1 \\ 0 & 1 & 1 \end{array}\right] \neq 1$$

   Since $f_{y'z'}$ is not a tautology, we conclude that f is not a tautology.

4. **[Validity of Expansion] (15 Points)**

   Problem 15 (Chap. 5), p. 211 in H&S.



a. $U = 6$, Cov = { }, MIS = $\{C1\}$, $L = 1$

b. $U = 6$, Cov = $\{x_1\}$, MIS = $\{C7\}$, $L = 2$

c. $U = 6$, Cov $= \{x_1, x_2\}$

$x_3 \equiv x_4 \equiv x_5 \Rightarrow x_4 = x_5 = 0$, x_3 essential

BestSol $= \{x_1, x_2, x_3\} \Rightarrow U = 3$

d. $U = 3$, Cov = $\{x_1\}$, MIS = $\{C7\}$, $L = 2$

e. $U = 3$, Cov $= \{x_1, x_3\}$

$x_4 \equiv x_5 \Rightarrow x_5 = 0$, x_4 essential

Sol = $\{x_1, x_3, x_4\}$, Cost(Sol) $= U \Rightarrow$ discard

f. $U = 3$, Cov = $\{x_1\}$, x_4 and x_5 essential

Sol = $\{x_1, x_4, x_5\}$, Cost(Sol) $= U \Rightarrow$ discard

g. $U = 3$, Cov = { }, MIS = $\{C1, C6\}$, $L = 2$

h. $U = 3$, Cov = $\{x_2\}$, MIS = $\{C4\}$, $L = 2$

i. $U = 3$, Cov $= \{x_2, x_3\}$

$x_4 \equiv x_5 \Rightarrow x_5 = 0$, x_4 essential

Sol = $\{x_2, x_3, x_4\}$, Cost(Sol) $= U \Rightarrow$ discard

j. $U = 3$, Cov = $\{x_2\}$, x_4 and x_5 essential

Sol = $\{x_2, x_4, x_5\}$, Cost(Sol) $= U \Rightarrow$ discard

k. $U = 3$, Cov = { }, x_3, x_4 and x_5 essential

Sol = $\{x_3, x_4, x_5\}$, Cost(Sol) $= U \Rightarrow$ discard

a.

	x_1	x_2	x_3	x_4	x_5
C1	1	1	1	0	0
C2	1	1	0	1	0
C3	1	1	0	0	1
C4	1	0	1	1	0
C5	1	0	1	0	1
C6	1	0	0	1	1
C7	0	1	1	1	0
C8	0	1	1	0	1
C9	0	1	0	1	1
C10	0	0	1	1	1

b.

	x_2	x_3	x_4	x_5
C7	1	1	1	0
C8	1	1	0	1
C9	1	0	1	1
C10	0	1	1	1

c.

	x_3	x_4	x_5
C10	1	1	1

d.

	x_3	x_4	x_5
C7	1	1	0
C8	1	0	1
C9	0	1	1
C10	1	1	1

e.

	x_4	x_5
C9	1	1

f.

	x_4	x_5
C7	1	0
C8	0	1
C9	1	1
C10	1	1

g.

	x_2	x_3	x_4	x_5
C1	1	1	0	0
C2	1	0	1	0
C3	1	0	0	1
C4	0	1	1	0
C5	0	1	0	1
C6	0	0	1	1
C7	1	1	1	0
C8	1	1	0	1
C9	1	0	1	1
C10	0	1	1	1

h.

	x_3	x_4	x_5
C4	1	1	0
C5	1	0	1
C6	0	1	1
C10	1	1	1

i.

	x_4	x_5
C6	1	1

j.

	x_4	x_5
C4	1	0
C5	0	1
C6	1	1
C10	1	1

g.

	x_3	x_4	x_5
C1	1	0	0
C2	0	1	0
C3	0	0	1
C4	1	1	0
C5	1	0	1
C6	0	1	1
C7	1	1	0
C8	1	0	1
C9	0	1	1
C10	1	1	1

Figure 1. **Solution to Probelm 1**

Solution:

$$
(\ F-\{c_i\}\ )_{\tilde{c}_i} = \left[\begin{array}{ccccc|c} v & w & x & y & z & f \\ - & - & 1 & 0 & 1 & 1 \\ - & 0 & 1 & - & - & 1 \\ 0 & - & 1 & - & 0 & 1 \\ 0 & - & - & - & 1 & 1 \\ 1 & 1 & 0 & 1 & - & 1 \end{array}\right]_{01---|1}
$$

$$
\Rightarrow \left[\begin{array}{ccccc|c} v & w & x & y & z & f_{\tilde{c}_i} \\ - & - & 1 & 0 & 1 & 1 \\ 0 & - & 1 & - & 0 & 1 \\ 0 & - & - & - & 1 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} x & y & z & f_{\tilde{c}_i} \\ 1 & 0 & 1 & 1 \\ 1 & - & 0 & 1 \\ - & - & 1 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} x & z & f_{\tilde{c}_i} \\ 1 & 0 & 1 \\ - & 1 & 1 \end{array}\right] \neq 1
$$

Since this cofactor is not tautologous, the expansion $00---|1 \rightarrow 0----|1$ is invalid.

5. **[Recursive Complementation] (15 Points)**

Problem 17 (Chap. 5), p. 212 in H&S. Is the cover of f' irredundant? Is it prime?

Solution:

$$
\begin{aligned}
(f)' &= (wx'y + w'xy + yz' + wxy' + wy'z')' \\
&= y(f_y)' + y'(f_{y'})' \\
&= y(wx' + w'x + z')' + y'(wx + wz')' \\
&= y[w(x' + z')' + w'(x + z')'] + y'[w(x + z')' + w'(0)'] \\
&= y[wxz + w'x'z] + y'[wx'z + w'] \\
&= w'x'yz + wxyz + w'y' + wx'y'z
\end{aligned}
$$

This cover of f' is not prime (as first and last cubes can be further expanded) and therefore it is not irredundant. It can be made a prime irredundant cover (indeed a minimal cover) by expanding its first and last cubes: $f' = w'x'z + wxyz + w'y' + x'y'z$.

6. **[Maximal Expansion] (20 Points)**

Determine all maximal expansions for each of the cubes in the on-set of this cover by:

a. Computing a cover for the off-set using recursive complementation

b. Constructing appropriate blocking matrices

Solving for all minimum cost solutions in the associated covering problemsSolution:

w	x	y	z	f
0	0	–	1	1
0	–	1	1	1
1	–	0	1	1
1	0	1	0	1
1	–	0	0	–
1	1	1	–	–
–	1	1	0	–

a. The off-set cover obtained by recursive complementation is:

$f' = w'x'z' + w'y'z' + w'xy'z + wx'yz$. Note that this cover is neither prime nor irredundant. This won't affect the answer to part c; it just causes larger blocking matrices.

b. and c. The blocking matrices and their minimal covers are:

	On-set Cubes												
Off-set Cubes	w'	x'	z	w'	y	z	w	y'	z	w	x'	y	z'
$w'x'z'$			1			1	1		1	1			
$w'y'z'$			1		1	1	1		1	1		1	
$w'xy'z$		1			1		1			1	1	1	1
$wx'yz$	1			1				1					1
Minimal cover:	$w'x'z$			$w'yz$			wy'			wz'			
Can be expanded?	No			No			Yes			Yes			