Print Name: Solutions

EECS 320

Exam I

This exam is closed book, a list of equations and useful constants is provided. More equations and constants are provided than you may need to use.

Show all work and ***state all assumptions*** to receive full credit. Use the space provided to answer questions. Assume room temperature unless stated otherwise.

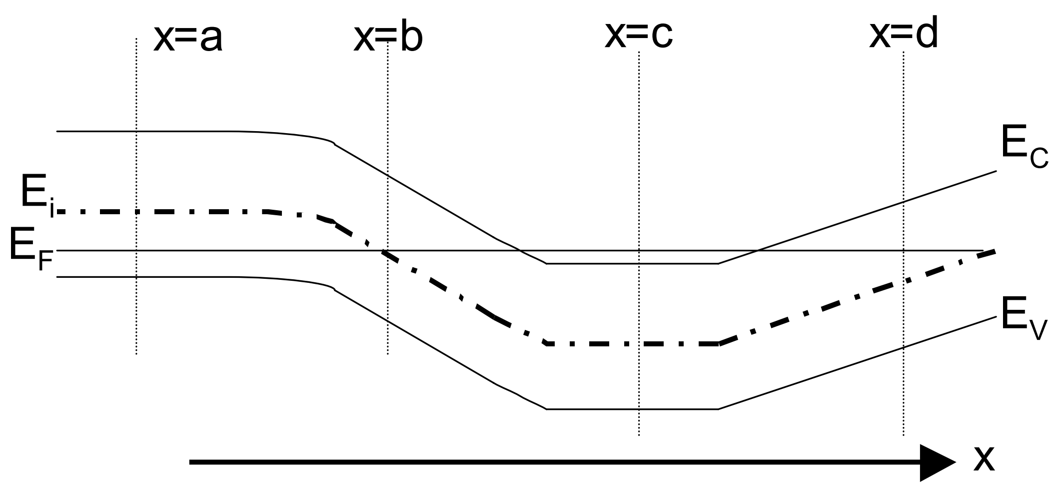
Matching, choose the *best* answer (2 points each)

____i____	1. Described why classical model for conductivity failed	a. Fermi-Dirac statistics
____a____	2. Describes the probability of occupying an electron state	b. mobility
____b____	3.Relates carrier velocity to electric field	c. density of states
____j____	4. Used to describe electron behavior in a semiconductor (versus a free electron)	d. drift
____c____	5. Describes how electron states are distributed in energy	e. Pauli exclusion principle
____h____	6. Process describing electron response to non-uniform electron concentration	f. holes
____k____	7. Method of altering conductivity in a semiconductor	g. Uncertainty principle
____d____	8. Process describing electron response to an electric field	h. diffusion
		i. Energy bands
		j. effective mass
		k. doping
		l. Einstein relation
		m. Boltzmann statistics

9. True or <u>False</u>: A material with high conductivity will always have high mobility . (2 points)

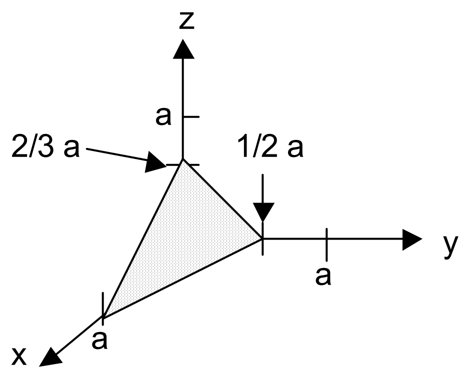
10. <u>True</u> or False: Fermi-Dirac statistics may always be used to describe electrons in a semiconductor. (2 points)

11. Study the equilibrium energy band diagram given below and circle the best choice for the following questions. (2 points each)



At x=a, the material is	n-type	or	p-type		
At x=a,	$n < n_i$	or	$n=n_i$	or	$n < n_i$
At x=b,	n<<p	or	n ~ p	or	n >> p
At x=b,	$J_n^{drift} < 0$	or	$J_n^{drift} = 0$	or	$J_n^{drift} > 0$
At x=b, the drift process forces electrons in the	+x direction	or	-x direction		
At x=c, the semiconductor is	degenerate	or	non-degenerate		
At x=c, electrons possess____ Kinetic energy than at x=a	less	or	greater		
At x=d,	$J_p^{diff} < 0$	or	$J_p^{diff} = 0$	or	$J_p^{diff} > 0$
At x=d,	$J_n^{total} < 0$	or	$J_n^{total} = 0$	or	$J_n^{total} > 0$

12. Find the Miller indices (hkl) for the following plane. (5 points)



Intercepts at a, a/2, 2a/3

Reciprocals are 1, 2, 3/2

Lowest whole number set is 2, 4, 3

Miller plane given by (243)

13. P-type and n-type regions of a semiconductor are joined together. Diffusion will clearly drive holes to the n-side and electrons to the p-side, however, no current flow is measured. Explain briefly in one sentence. (8 points)

The diffusion process leaves behind ionized dopants. This fixed charge sets up an electric field resulting in a drift current. In equilibrium, the drift and diffusion components will balance out to a net current of zero.

14. A typical silicon microprocessor is heated to 200^0C, and no longer appears to be working. After cooling to room temperature, the computer chip appears to be working fine again. What do you suspect to be the cause for this behavior? (7 points)

The microprocessor uses dopants to tailor conductivity in specified regions, with properties ranging form insulating to conducting over several orders of magnitude in conductivity. When the microprocessor is heated, the intrinsic carrier concentration becomes comparable to the doping concentrations, the conductivity is no longer doping dependent and everything looks conducting: the devices no longer work as designed. When the device is cooled, the intrinsic carrier concentration returns to low levels and the microprocessor works again.

15. A uniformly doped region of n-type silicon (N_D=10^{16} cm^{-3}) of length L=1μm is illuminated uniformly with G=10^{21} $cm^{-3}s^{-1}$. The material has a minority carrier lifetime of τ=1μs and mobility of 500 cm^2/Vs, and we may assume that R-G is negligible. We know the boundary conditions for minority carrier density, 5x10^{10} cm^{-3} at x=0, and 0 at x=L.

Write the simplified minority carrier diffusion equation needed to determine the excess minority carrier density in steady state. (4 points)

$$\frac{\partial \Delta p_n}{\partial t} = D_p \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} + G$$

steady state, no R-G, results in simplified expression:

$$0 = D_p \frac{\partial^2 \Delta p_n}{\partial x^2} + G$$

Solve the minority carrier diffusion equation and write an expression for the excess minority carrier density as a function of position . (10 points)

$\frac{\partial^2 \Delta p_n}{\partial x^2} = -\frac{G}{D_p}$, integrate twice to get excess hole density

$\Delta p_n = -\frac{G}{2D_p} x^2 + ax + b$, find a and b using boundary conditions

$$\Delta p_n(0) = b$$

$$\Delta p_n(L) = -\frac{G}{2D_p} L^2 + aL + \Delta p_n(0)$$

$$a = \frac{GL}{2D_p} - \frac{\Delta p_n(0)}{L}$$

$$\Delta p_n = -\frac{G}{2D_p} x^2 + \left[\frac{GL}{2D_p} - \frac{\Delta p_n(0)}{L}\right] x + \Delta p_n(0)$$

$$\Delta p_n = \frac{G}{2D_p}\left(Lx - x^2\right) + \Delta p_n(0)\left(1 - \frac{x}{L}\right)$$

Calculate the excess minority carrier density at x=L/2. (5 points)

$$\Delta p_n\left(\frac{L}{2}\right) = \frac{G}{2D_p}\left(\frac{L^2}{2} - \frac{L^2}{4}\right) + \Delta p_n(0)\left(1 - \frac{1}{2}\right) = \frac{G}{2D_p}\frac{L^2}{4} + \frac{\Delta p_n(0)}{2}$$

$$\Delta p_n\left(\frac{L}{2}\right) = \frac{G}{2D_p}\frac{L^2}{4} + \frac{\Delta p_n(0)}{2}$$, from Einstein relation, D_p=13 cm^2/s

$$\Delta p_n\left(\frac{L}{2}\right) = \frac{10^{21}}{2(13)}\frac{\left(10^{-4}\right)^2}{4} + \frac{5\times10^{10}}{2} = 1.21\times10^{11}cm^{-3}$$

Compare the minority carrier diffusion length to the length of the silicon region of interest (L). Is the assumption that R-G is negligible valid? (5 points)

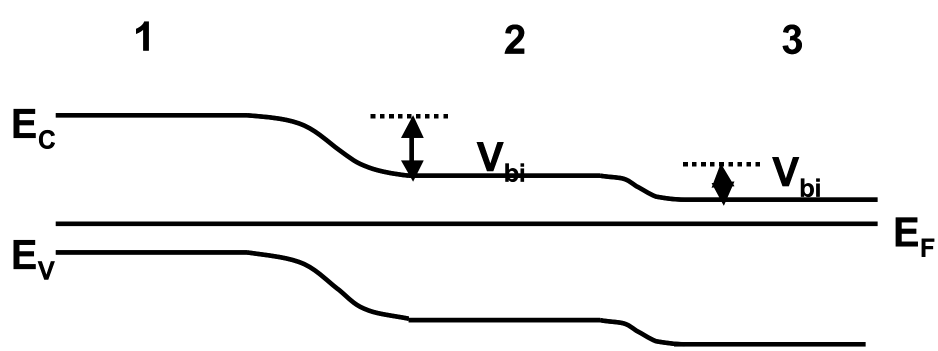
$L_p=(D_p\tau_p)^{1/2}=(13\text{x}10^{-6})^{1/2}=3.6\text{x}10^{-3}$ cm = 36μm

L_p is much larger than L, recombination is insignificant in L, assumption valid

16. A piece of silicon contains three distinct regions of varying doping, as shown in the following. In each region, the doping is uniform and changes abruptly at the interface with the next region. You may assume that the regions are significantly longer than the minority carrier diffusion length.

Region 1	Region 2	Region 3
$N_A=10^{17}$ cm^{-3}	$N_D=10^{15}$ cm^{-3}	$N_D=5\text{x}10^{17}$ cm^{-3}

a) Sketch the energy band diagram for this structure in equilibrium at room temperature. Indicate E_C, E_V, and E_F. (5 points)



b) Calculate the built-in voltage between Region 1 and Region 2. (5 points).

$$V_{bi} = \frac{kT}{q}\ln\left(\frac{N_D N_A}{n_i^2}\right) = 0.026\ln\left(\frac{10^{15}\times 10^{17}}{\left(10^{10}\right)^2}\right) = 0.72V$$

c) There appears to be a built-in voltage between Region 2 and Region 3. Determine this built-in voltage, noting you cannot directly apply the V_{bi} equation for a p-n junction. (Hint: examine the equilibrium Fermi level position). (8 points)

The Fermi level in region 3 sits close to the conduction band. The built-in voltage is the difference between E_C-E_F for region 2 and region 3.

Region 1

$$E_C - E_f = -kT\ln\left(\frac{n}{N_C}\right) = -0.026\ln\left(\frac{10^{15}}{2.78\times10^{19}}\right) = 0.27eV$$

Region 2

$$E_C - E_f = -kT\ln\left(\frac{n}{N_C}\right) = -0.026\ln\left(\frac{5\times10^{17}}{2.78\times10^{19}}\right) = 0.10eV$$

qV_{bi}=0.27-0.10=0.17eV

V_{bi}=0.17V

EECS 320 Midterm Exam I equation sheet

$$h = 6.626\times10^{-34} \text{ J-s}$$
$$\hbar = 1.055\times10^{-34} \text{ J-s}$$
$$q = 1.602\times10^{-19} \text{ C}$$
$$eV = 1.602\times10^{-19} \text{ J}$$
$$m_0 = 9.11\times10^{-31} \text{ kg}$$
$$k = 8.617\times10^{-5} \text{ eV/K}$$
$$E = \frac{\hbar^2 k^2}{2m^*} + V_0$$
$$np = n_i^2$$
$$n - N_d^+ - p + N_a^- = 0$$
$$f(E) = \frac{1}{1+\exp\left(\frac{E-E_f}{kT}\right)}$$
$$f(E) = \exp\left(-\frac{E-E_f}{kT}\right)$$
$$g_C(E) = \frac{\sqrt{2}m_n^{*3/2}(E-E_C)^{1/2}}{\pi^2\hbar^3}$$
$$g_V(E) = \frac{\sqrt{2}m_p^{*3/2}(E_V-E)^{1/2}}{\pi^2\hbar^3}$$
$$F = -\frac{dV}{dx} = \frac{1}{q}\frac{dE}{dx}$$
$$E_f = E_C + kT\ln\left(\frac{n}{N_C}\right)$$
$$E_f = E_V - kT\ln\left(\frac{p}{N_V}\right)$$
$$E_f = E_C + kT\left[\ln\left(\frac{n}{N_C}\right) + \frac{1}{\sqrt{8}}\frac{n}{N_C}\right]$$
$$E_f = E_V - kT\left[\ln\left(\frac{p}{N_V}\right) + \frac{1}{\sqrt{8}}\frac{p}{N_V}\right]$$

$$N_C = 2\left(\frac{m_n^* kT}{2\pi\hbar^2}\right)^{3/2}$$
$$N_V = 2\left(\frac{m_p^* kT}{2\pi\hbar^2}\right)^{3/2}$$
$$np = 4\left(\frac{kT}{2\pi\hbar^2}\right)^3 \left(m_n^* m_p^*\right)^{3/2} \exp\left(-\frac{E_G}{kT}\right)$$
$$n_i = 2\left(\frac{kT}{2\pi\hbar^2}\right)^{3/2} \left(m_n^* m_p^*\right)^{3/4} \exp\left(-\frac{E_G}{2kT}\right)$$
$$n_i = \sqrt{N_C N_V}\exp\left(-\frac{E_G}{2kT}\right)$$
$$E_i = \frac{E_C+E_V}{2} + \frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right)$$
$$n = \int g_C(E) f(E)\, dE$$
$$p = \int g_V(E)(1-f(E))\, dE$$
$$n = n_i\exp\left(\frac{E_f-E_i}{kT}\right)$$
$$n = N_C\exp\left(\frac{E_f-E_C}{kT}\right)$$
$$p = n_i\exp\left(-\frac{E_f-E_i}{kT}\right)$$
$$p = N_V\exp\left(\frac{E_V-E_f}{kT}\right)$$
$$n = \frac{N_D-N_A}{2} + \sqrt{\left(\frac{N_D-N_A}{2}\right)^2 + n_i^2}$$
$$p = \frac{N_A-N_D}{2} + \sqrt{\left(\frac{N_A-N_D}{2}\right)^2 + n_i^2}$$
$$\sigma = nq\mu_n + pq\mu_p$$
$$\mu = \frac{e\tau_{sc}}{m^*}$$

$$\frac{D}{\mu} = \frac{kT}{q}$$

$$J_n = q\mu_n nF + qD_n \frac{dn}{dx}$$

$$J_p = q\mu_p pF - qD_p \frac{dp}{dx}$$

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} + (G - R)$$

$$\frac{\partial \Delta n_p}{\partial t} = D_n \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n} + G$$

$$\frac{\partial \Delta p_n}{\partial t} = D_p \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} + G$$

$$L_N = \sqrt{D_N \tau_n}$$

$$L_P = \sqrt{D_P \tau_p}$$

$$V_{bi} = \frac{kT}{q}\ln\left(\frac{N_D N_A}{n_i^2}\right)$$

$$W = \sqrt{\frac{2\varepsilon V_{bi}}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)}$$

$$n_i = 2.5\times10^{19}\left(\frac{m_n^*}{m_0}\frac{m_p^*}{m_0}\right)^{3/4}\left(\frac{T}{300}\right)^{3/2}\exp\left(-\frac{E_G}{2kT}\right)\text{cm}^{-3}$$

$$N_C = 2.5\times10^{19}\left(\frac{m_n^*}{m_0}\right)^{3/2}\left(\frac{T}{300}\right)^{3/2}\ \text{cm}^{-3}$$

$$N_V = 2.5\times10^{19}\left(\frac{m_p^*}{m_0}\right)^{3/2}\left(\frac{T}{300}\right)^{3/2}\ \text{cm}^{-3}$$

Table of properties of selected semiconductors (at 300K)

Property	Si	Ge	GaAs
N_C (cm^{-3})	2.78×10^{19}	1.04×10^{19}	4.45×10^{17}
N_V (cm^{-3})	9.84×10^{18}	6.0×10^{18}	7.72×10^{18}
n_i (cm^{-3})	10^{10}	2.3×10^{13}	1.8×10^{6}
E_G (eV)	1.12	0.66	1.42
m_n^*/m_0	1.18	0.55	0.067
m_p^*/m_0	0.81	0.36	0.52
ε_r	11.8	16	13.1
χ (eV)	4.05	4.0	4.07